State Estimation of Distribution Networks

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Declaration

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Statement of Co-Authorship

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Paper 4, *< An Observability Index for Distribution Networks using Information Entropy >:*
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Paper 6, *< Robust state estimation in distribution networks >:*
Located in Chapter 2
Candidate was the primary author (60%), Author 1 (40%) contributed to the idea, its formalisation and development.

Paper 7, *< A Practical Approach to Meter Placement in Distribution Networks >:*
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Preface

Abstract

Power system state estimation has been introduced over four decades ago and since then has become an integral part of the day to day network operation. State estimators are implemented in almost every control center around the world and are used to continuously monitor the power system in a near real-time fashion. More specifically, the state estimation algorithm is used to obtain an estimate of the network state while acting as a filter for measurement errors by utilizing the redundant nature of the available measurement set. Over time, a large number of redundant measurement devices were installed throughout transmission networks, which makes it possible to estimate the state of a transmission system with a high degree of accuracy. However, this is not the case in distribution networks where only a very small number of real-time measurements are available. In absence of real-time measurements, pseudo-measurements are normally used in order to enable an application of the state estimation method. Pseudo-measurements are forecasted values for loads and/or generation connected at a specific point in the network which generally have large margins of error associated with them. This means that if a large number of pseudo-measurements are used to estimate a network state, the resulting state may be significantly different from the actual network state. This is the reason why state estimation has to this day not been widely implemented in distribution networks.

In recent years, however, the growing amount of renewable generation connected at the distribution level has resulted in an increased risk that network constraints are violated. In order to perform a security assessment or take required control ac-

tions, the network operator must obtain reliable information about the state of the distribution network. This has given rise to the need of implementing state estimation in distribution networks in order to obtain the required real-time information about the networks state.

Before the state estimation can be performed, it has to be determined if a unique estimate of the network state can be obtained from the available set of measurements. This is done by the observability analysis. However, traditional methods only determine if a state can be calculated and not if this result can provide practical information to the distribution network operator. Hence, a new probabilistic approach to observability is developed in order to overcome this limitation. The developed method assesses the network observability depending on the accuracy of the estimated network state and the proximity of the estimated parameters to their constraints under worst case consideration.

In case the uncertainty in an estimated state is too large to be practical, additional measurement devices have to be placed in order to improve the accuracy. However, due to economic constraints, only a small number of buses can be equipped with real-time measurements. This makes optimal meter placement an important tool for the implementation of state estimation in distribution networks. As part of this thesis a new meter placement method is described which can potentially reduce the number of required measurement devices compared to conventional meter placement methods while providing a practical level of state estimation accuracy.

The goal of this research is to develop new methods that support the application of state estimation in distribution networks in presence of large uncertainties in the state estimation inputs due to the lack of real-time measurements. For this purpose the focus of this thesis is on the accuracy of the state estimation. In particular the aspects of observability, uncertainty quantification, meter placement and the practical representation of state estimation results have been considered.

Thesis Outline

This thesis is organized into five chapters and the following gives a brief overview of the chapter contents:

- **Chapter 1** provides a general introduction into the topic of state estimation, and its relevance to distribution networks. It also highlights the differences between distribution networks and the transmission systems which make an application of the traditional approach to state estimation in distinction networks challenging.

- **Chapter 2** focuses on the state estimation process. Here, a probabilistic approach to state estimation which is based on confidence is described which is suitable for distribution networks with a low number of measurement devices. This approach combines information about the state estimation accuracy and the expected values of the estimated parameters into a single number representation. This is achieved by calculating the confidence that the estimated parameters are within their respective constraints. The proposed approach makes it possible to objectively assess the state estimation result and quickly identify critical areas even if a large amount of uncertainty is present in the state estimation result. This chapter also describes a new method to calculate the probability density functions of the estimated parameters in real-time, as well as a method that determines if three-phase state estimation has to be used based on how accurate the state estimation may take place considering the level of unbalance in the system.

- **Chapter 3** introduces a probabilistic approach to observability. This approach is based on practical considerations and is designed for an application in distribution networks. It assesses the accuracy of the state estimation result under worst case consideration in order to identify critical areas in the network. This chapter also describes an index for the distribution network observability which provides a single continues number that gives an indication of the network observability. Here the concept of information entropy which was developed by Shannon in 1948 is used to quantify the uncertainty in a state estimation result.

- In **Chapter 4** a new meter placement method is developed which is based on previous findings and practical considerations. Conventional meter placement methods normally aim to increase the accuracy of the estimated network state to a predefined and uniform level. The method presented in this chapter on the other hand only increases the accuracy of critical parameter estimates. This results in a smaller number of required measurement devices while maintaining a practical level of accuracy.

- **Chapter 5** provides the conclusion and points out potential directions for future research.

Supporting Publications

Book Chapter:

1. B. Brinkmann, M. Negnevitsky, "Probabilistic State Estimation in Distribution Networks", in Advances in Electric Power and Energy Systems: Power Systems Engineering, Wiley-IEEE Press, Forthcoming.

Journal papers:

1. B. Brinkmann and M. Negnevitsky, "A Probabilistic Approach to Observability of Distribution Networks," IEEE Trans. on Power Syst., pp. 1-10, 2016.

2. B. Brinkmann and M. Negnevitsky, "A Practical Approach to State Estimation in Distribution Network " IEEE Trans. on Power Distribution, pp. 1-8 (under review), 2017.

Conference Papers:

1. B. Brinkmann, M. Negnevitsky, T. Yee, and T. Nguyen, "An Observability Index for Distribution Networks using Information Entropy," in Australasian Universities Power Engineering Conf. (AUPEC), Wollongong, NSW, 2015, pp. 1-6.

2. B. Brinkmann and M. Negnevitsky, "A Practical Approach to Observability Analysis and State Estimation in Distribution Networks," in IEEE Power and Energy Society General Meeting, Boston, MA, 2016, pp. 1-5.

3. B. Brinkmann and M. Negnevitsky, "Robust state estimation in distribution networks," in Australasian Universities Power Engineering Conference (AUPEC), 2016, pp. 1-5.

4. B. Brinkmann, R. Scott, K. Bicevskis, and M. Negnevitsky, "A Practical Approach to Meter Placement in Distribution Networks," in PowerTech 2017, Manchester, UK, 2017 pp. 1-6.

5. B. Brinkmann, K. Bicevskis, R. Scott, and M. Negnevitsky, "Evaluation of Single- and Three-Phase State Estimation in Distribution Networks," in AUPEC, Melbourne, 2017 pp. 1-5.

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Chapter 1

Introduction

1.1 Background

State estimation refers to the process of estimating the network state from the available set of measurements. The idea of using state estimation in power systems was first proposed by Schweppe in 1970 [1]–[3]. It utilizes redundant measurements to filter out measurement errors and to find the most likely state of a network. The result of the state estimation process is the state vector which uniquely identifies the state of a network. If we consider a network which has a number of n buses, the state vector consists of $2n$ parameters (voltage magnitudes and angles at every bus). However, the voltage angle at one particular bus is used as a reference and is generally set to zero ($\theta_1 = 0$) [4]. Therefore, the state vector is defined by

$$\boldsymbol{x}^T = [\theta_2, \ldots, \theta_n, V_1, \ldots, V_n] \tag{1.1}$$

where θ referrers to the voltage angle, and V to the voltage magnitude. The measurement data used in the state estimation process is provided by the supervisory control and data acquisition (SCADA) system. The SCADA system is responsible for the system wide acquisition of measurement data and works by aggregating data from local measurements at Remote Terminal Units (RTUs) which then transmit this information to a central server. Measurements connected to RTUs can be subdivided in to two groups; analog measurements such as voltage and current magnitudes, and digital measurements such as the status of circuit breakers, switches, and transformer taps. An overview of the SCADA system is shown in Fig. 1.1.

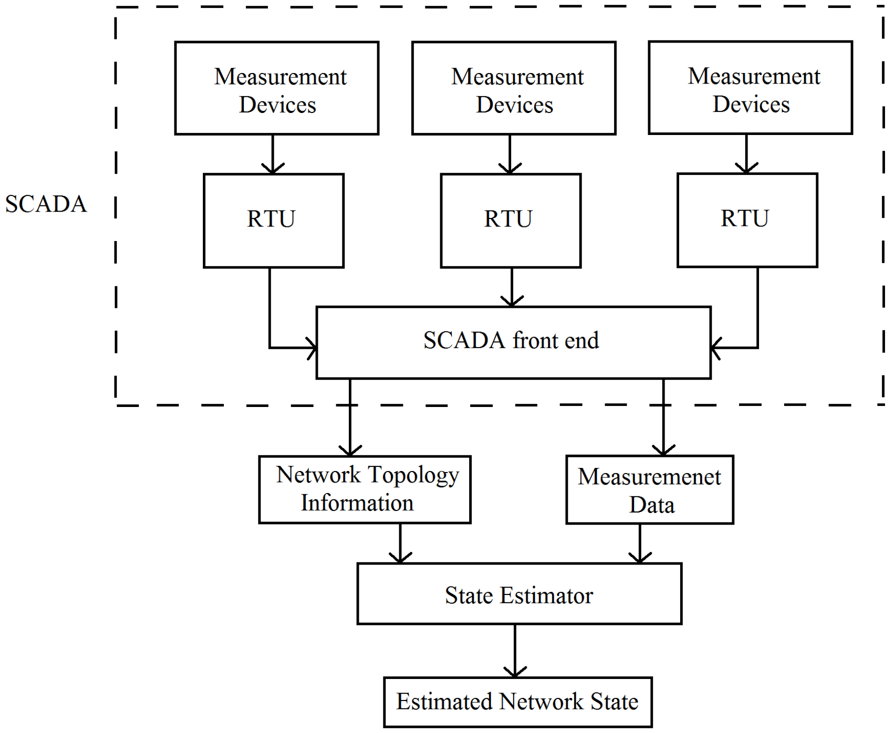

Figure 1.1: State estimation SCADA block diagram

The data collected by the SCADA system is provided to all network functionalities that require it, which includes the state estimation process. In addition to the measurement data provided by the SCADA system, a model of the network is also required to solve the state estimation problem. The network model is formed by a set of nodal equations which are created by using either the nodal analysis method based on the Kirchhoff's current law, or by using the mesh analysis which is based on the Kirchhoff's voltage law. However, the nodal analysis is used almost exclusively due to the following advantages:

- The number of equations is usually smaller compared to the mesh approach
- Non-planar circuits do not represent a problem
- Parallel branches do not increase the number of equations

- It is strait forward to represent transformers with off-nominal tap settings

The current between two nodes is represented by the nodal voltages and the branch admittance.

$$i_{ij} = y_{ij}(V_i - V_j) \tag{1.2}$$

By applying the Kirchhoffs current law it is possible to represent the current injections at each bus in the network by the following equations:

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1N} \\ y_{21} & y_{22} & \cdots & y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N1} & y_{N2} & \cdots & y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \boldsymbol{Y} \cdot \boldsymbol{V} \tag{1.3}$$

where i_k is the current injection at Bus k, V_k is the voltage at Bus k, N is the number of nodes in the network, y_{ij} is the admittance between Bus i and Bus j, and y_{ii} is the self admittance of the node i given by

$$y_{ii} = \sum_{q=1 \neq i}^{N} y_{iq} \tag{1.4}$$

The matrix Y is generally referred to as the bus admittance matrix. Note that generally currents or power-flows that enter a node are considered to be positive and currents or power-flows exiting a node are negative. Transformers with off nominal tap settings can be integrated into the Y matrix by using the following model

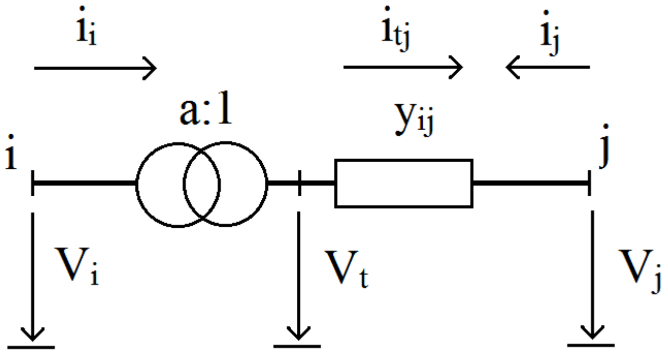

Figure 1.2: Transformer Model

where a is the tap ratio of the transformer and y_t is the series admittance of the transformer. The transformer between Bus i and Bus t is assumed to be ideal, and therefore, has a series admittance equal to zero. Now the following relationships between Bus t and Bus j can be defined.

$$\begin{bmatrix} i_{tj} \\ i_j \end{bmatrix} = \begin{bmatrix} y_t & -y_t \\ -y_t & y_t \end{bmatrix} \begin{bmatrix} V_t \\ V_j \end{bmatrix} \tag{1.5}$$

However, in order to represent the relationship between Bus i and Bus j the tap ratio of the transformer has to be accounted for. This is achieved by considering the following relationships and substituting them into the equation (1.5).

$$i_{tj} = a \cdot i_i \tag{1.6}$$

$$V_t = \frac{V_i}{a} \tag{1.7}$$

After incorporating these relationships into the equation (1.5), the final transformer model is given by

$$\begin{bmatrix} i_i \\ i_j \end{bmatrix} = \begin{bmatrix} y_t/a^2 & -y_t/a \\ -y_t/a & y_t \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{1.8}$$

In order to integrate this model into the bus admittance matrix Y the following modifications have to be made

$$
\begin{aligned}
Y_{kk}^{new} &= Y_{kk} + y_t/a^2 \\
Y_{km}^{new} &= Y_{km} - y_t/a \\
Y_{mk}^{new} &= Y_{mk} - y_t/a \\
Y_{mm}^{new} &= Y_{mm} + y_t
\end{aligned}
\tag{1.9}
$$

The most commonly used approach to calculate the network state is the Newton Raphson method. In this method, buses are classified into one out of three different bus types. Namely the voltage controlled bus, the load bus and the slack bus. The voltage controlled bus is used to represent generators, where it is assumed that the output power P is known and the voltage at the connection point is controlled by adjusting the reactive power output. Hence, for this bus type the injected active power P and the voltage magnitude V have to be specified. Next is the P-Q bus represents a known amount of active and reactive power injected at a specific bus. Therefore, P and Q have to be specified for this bus. Every network requires a bus which provides a reference for the voltage magnitude and voltage angle. This bus is commonly referred to as the slack bus or swing bus. Furthermore, since the losses in the system are not known, the load and generation cannot be specified at every bus. Hence, the slack bus also compensates for the loses in the system, and therefore, has the voltage magnitude and angle specified but not the active power P and the reactive power Q. This information is related to the state vector $\boldsymbol{x}$ by the vector of non-linear measurement functions $\boldsymbol{h}(\boldsymbol{x})$

$$
\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{e} \tag{1.10}
$$

where $\boldsymbol{z}$ is the vector that contains all measurements used in the state estimation, and $\boldsymbol{e}$ is the vector of measurement errors. In order to solve the system of non-linear equations the Newton-Raphson method uses an iterative process which starts by making an initial guess of the network state which is reasonably close to the actual network state. For this purpose generally all voltage magnitudes are set to one per unit and all voltage angles are set to zero which is also referred to as a flat start. The problem is then solved iteratively by

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - \boldsymbol{H}(\boldsymbol{x}^k)[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}^k)] \tag{1.11}$$

where k is the iteration index and $\boldsymbol{H}$ is the Jacobian matrix which is defined as

$$\boldsymbol{H} = \left[\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right] = \begin{bmatrix} 0 & \frac{\partial V_i}{\partial V} \\ \frac{\partial P_i}{\partial \theta} & \frac{\partial P_i}{\partial V} \\ \frac{\partial P_{ij}}{\partial \theta} & \frac{\partial P_{ij}}{\partial V} \\ \frac{\partial Q_i}{\partial \theta} & \frac{\partial Q_i}{\partial V} \\ \frac{\partial Q_{ij}}{\partial \theta} & \frac{\partial Q_{ij}}{\partial V} \end{bmatrix} \tag{1.12}$$

1.2 Observability

Before the state estimation can be performed, it has to be determined if a unique estimate of the network state can be obtained from the available set of measurements [5]. This is done by the observability analysis. In order to calculate the state of a network, a number of linearly independent measurements equal to or greater than the number of parameters that describe the network state have to be available [6]. Hence, for a network with a number of N nodes a number of $2N - 1$ linearly independent measurements has to be available. Here the -1 represents the fact that the angle at the slack bus is used as a reference, and therefore, does not have to be estimated. A network that fulfills this criterion is considered to be observable. If the number of linearly independent measurements is not sufficient to obtain a unique solution for the network state, the network is considered to be unobservable. The observability of a network can be linked to the column rank of its Jacobian matrix [7], [8]. The rank of the Jacobian matrix can be found by for instance using a Gaussian elimination or the Cholesky decomposition which provide the lower and upper triangular matrix form of the Jacobian matrix. The rank of the Jacobian matrix is equal to the number of non zero elements in the upper triangular matrix of the Jacobian matrix [9]. The observability analysis can be executed off-line and on-line. The off-line observability analysis determines if the available set of measurements is sufficient for the state estimation to be performed. If this is not the case additional measurements have to be added until the network becomes observable. An on-line application of the observability analysis ensures that an estimate of the network state can be obtained form the current set of measurements received from the SCADA system. This is necessary since communication errors, topology changes, or failed measurements could lead to un-observability. A lack of real-time measurements can always be compensated by the use of pseudo-measurements [3], [8], [10]. Pseudo-measurements are forecasted values for loads and/or generation connected at a specific node which are treated as random variables with an appropriate expected value and probability distribution. There are multiple ways how pseudo-measurements can be generated. For instance by using typical load profiles and historical billing information of a customer. A typical load profile for a residential load in Tasmania, Australia during Winter is shown in Fig. 1.3.

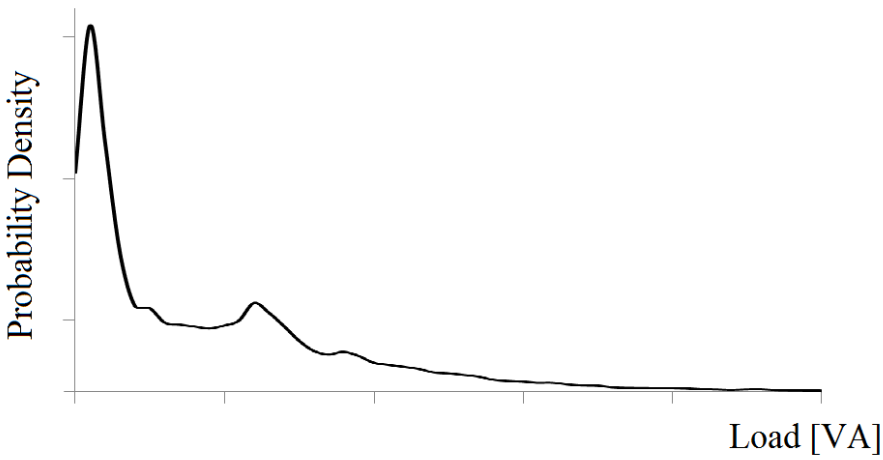

Figure 1.3: Typical probability density function of a residential load

The load on a transformer which supplies a number of different customers can be modeled by using the load synthesis of the connected customers which in term are represented by their respective typical load profiles. Loads can generally be subdivided into three categories (commercial, residential, and industrial) where each load type has a different typical load profile. In reality pseudo-measurements are also often modeled by assuming Gaussian distribution since this is compatible with the weighted least squares method which is the most popular state estimation method. It is, also important to note that since pseudo-measurements are based on historical data, they generally have large margins of error associated with them. This is due to the fact that it is not possible to accurately predict loads and/or generation from historical data.

1.3 State Estimation

If the number of available measurements is greater then the number of elements in the state vector, the network is referred to as overdetermined. This means that more measurements are available as are required to calculate the network state. However, due to the unavoidable measurement errors, redundant measurements will generally not agree with each other. This which means that the most likely combination of measurement values has to be found. This task is fulfilled by the state estimator which uses a maximum likelihood estimation. The aim of the maximum likelihood estimation is to find the most likely estimate of a network state given the measurement values and their associated accuracies. The measurements provided by the SCADA system are treated as random variables with known probability density functions that provide information about their respective accuracies. The joint probability distribution of the measurements is referred to as the likelihood function. Assuming that the measurements in $\boldsymbol{z}$ are independent of each other, the likelihood function of the measurement vector $\boldsymbol{z}$ is given by

$$PDF_m(\boldsymbol{z}) = PDF(z_1) \times PDF(z_2) \times \cdots \times PDF(z_m) \tag{1.13}$$

where $\boldsymbol{z}^T = [z_1, z_2, \ldots, z_m]$ and m is the number of available measurements. This function has its maximum value when the estimated values are closest to their actual values. Generally measurement errors are assumed to follow a Gaussian distribution which means that the probability density function of a measurement is defined by

$$PDF(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} \tag{1.14}$$

where z is a random variable that represents the measurement, μ is the mean or expected value of the measurement z and the standard deviation of z is given by σ.

The Maximum likelihood estimation attempts to find the combination of measurement values for which $PDF_m(\boldsymbol{z})$ is maximum. This leads to the formulation of the weighted least squares appraoch which is the most commonly used method to solve the state estimation problem. Some state estimation methods, such as the weighted least absolute value and the Schweppe Huber generalized M estimator have been successfully applied to transmission networks. However, they are unable to provide accurate and consistent results in distribution networks as demonstrated in [11]. The weighted least squares approach uses the log-likelihood function of $\boldsymbol{z}$ given by

$$\log(PDF_m(\boldsymbol{z})) = -\frac{1}{2}\sum_{i=1}^{m}\left(\frac{z_i-\mu_i}{\sigma_i}\right)^2 - \frac{m}{2}\log(2\pi) - \sum_{i=1}^{m}\log(\sigma_i) \tag{1.15}$$

The values of μ_i and σ_i are provided by the SCADA system and stay constant during the estimation process. The objective of the wighted least squares method is to minimize the following function in order to maximize (1.15)

$$\text{minimize} \quad \sum_{i=1}^{m}\left(\frac{z_i-\mu_i}{\sigma_i}\right)^2 \tag{1.16}$$

where μ_i represents the expected value ($E(z_i)$) of the measurement i. The difference between z_i and μ_i is also referred to as the measurement residuals (r_i) and the inverse of the measurement variance is known as the measurement weight ($\sigma_i^{-2} = W_i$). This leads to the following formulation of the optimization problem.

$$\text{minimize} \quad \sum_{i=1}^{m} W_i \times r_i^2 \tag{1.17}$$

$$\text{subject to} \quad r_i = z_i - h_i(x) \tag{1.18}$$

where $h_i(x)$ is a non linear function that relates the state vector to the measurement i [12]. The weighted least squares state estimator minimizes the following objective function

$$J(x) = \left[\sum_{i=1}^{m} \frac{(\boldsymbol{z}_i - \boldsymbol{h}_i(\boldsymbol{x}))^2}{\boldsymbol{R}_{ii}} \right] \tag{1.19}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{z}$ is the measurement vector, m is the number of measurements used in the state estimation, $\boldsymbol{R}$ is a diagonal matrix containing the variance of the measurements with

$$\boldsymbol{R} = diag[\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2]. \tag{1.20}$$

Hence $\boldsymbol{R}_{ii}$ donates the standard deviation of measurement i. It can be seen that the sum of the squared differences between the measurements and the estimated values, relative to the standard deviation of the measurement errors is minimized. Equation (1.19) can be rewritten in matrix form as

$$J(\boldsymbol{x}) = [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})]^T \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] \tag{1.21}$$

Since the aim is to minimize the objective function $J(\boldsymbol{x})$, the first directive of (1.21) has to be equal to zero if evaluated at the optimal solution.

$$\begin{aligned} \frac{\partial J(\boldsymbol{x})}{\partial \boldsymbol{x}} &= -\boldsymbol{H}^T(\boldsymbol{x}) \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] = 0 \\ \text{where } \boldsymbol{H}(\boldsymbol{x}) &= \frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}} \end{aligned} \tag{1.22}$$

Since (1.21) is a nonlinear function, the solution has to be found iteratively. In each iteration the flowing expression is evaluated.

$$\triangle \boldsymbol{x}^{k+1} = \boldsymbol{H}^T(\boldsymbol{x}^k) \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}^k)] \boldsymbol{G}^{-1}(\boldsymbol{x}^k) \tag{1.23}$$

where k represents the iteration index, $\boldsymbol{G}(\boldsymbol{x}^k)$ is called the gain matrix which is defined as $\boldsymbol{H}^T(\boldsymbol{x}^k) \boldsymbol{R}^{-1} \boldsymbol{H}(\boldsymbol{x}^k)$, and $\triangle \boldsymbol{x}^{k+1} = (\boldsymbol{x}^{k+1} - \boldsymbol{x}^k)$. The solution is found ones $\triangle \boldsymbol{x}^{k+1}$ is below a predefined threshold. Historically distribution networks have been operated in a passive fashion, where all control strategies are derived off-line during the network planning phase. This has worked well in the past where the purpose of distribution systems has been to transport electric power in one direction only, from the transmission substations to the consumer. However, in recent years we have experienced a significant increase in renewable generation. Furthermore, this renewable generation is often connected at the distribution level. This is emphasized by the fact that in Queensland (Australia) 29.6% of all dwellings are equipped with a photovoltaic system [13]. The total generation capacity of photovoltaic installations in Australia has been constantly increasing in a linear fashion since 2010 as shown in Fig. 1.4.

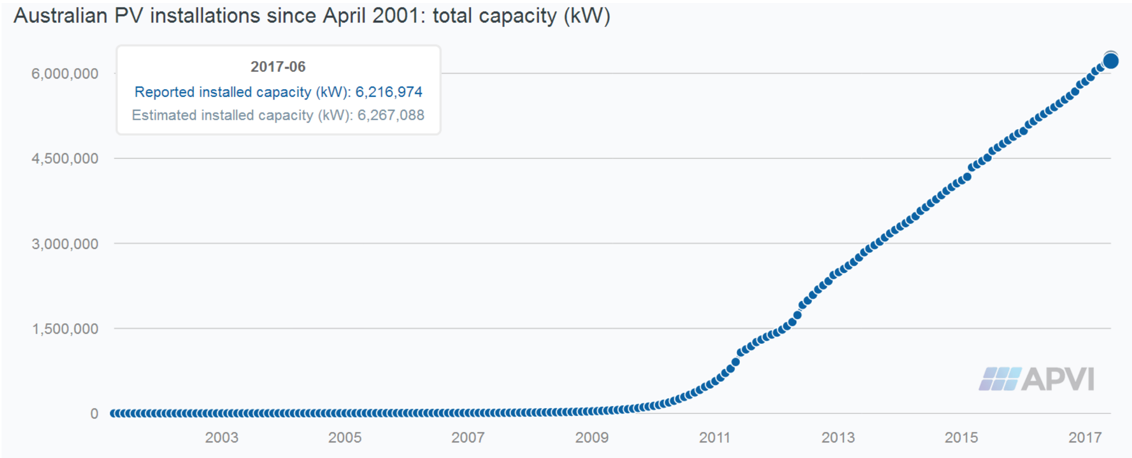

Figure 1.4: Installed Solar Capacity in Australia [14]

Due to the growing amount of renewable generation connected at the consumer level, the flow of power can become reversed. This can result in an increased risk of violating network constraints [15]–[17].The constraints of a network consists of physical system limitations such as the thermal limits of the individual network components as well as regulations such as the voltage compliance range. Before the introduction of distributed generation, only the case where the load is at its maximum expected value had to be considered in order to design a distribution networks and to develop control strategies. However, with the increase of distributed generation in distribution systems, another scenario has to be considered which is the case of minimum load maximum generation. These cases are refereed to as the a worst case scenarios which represent the network states for which it is most likely that network constraints are violated. To illustrate the effect of distributed generation on a distribution feeder, the voltage profiles for the case of minimum load and maximum generation as well as the maximum load no distributed generation have been illustrated in Fig. 1.5. The six bus feeder has a uniform line length and an equal loading between the individual buses.

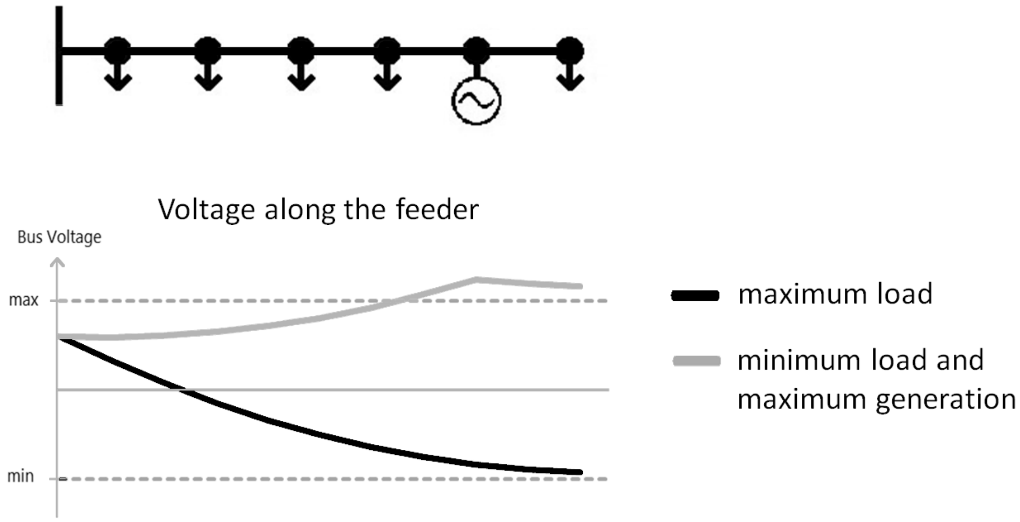

Figure 1.5: Voltage Rise Effect

The primary goal of a network operator is to maintain a system state that is normal and secure. A network state is referred to as normal if all loads are supplied with power and the system operates within its constraints. The constraints of a network are represented by either the physical limits of the system such as the thermal limit of a transformer, or regulations such as the voltage limits. A network is regarded as secure if no contingency event from a number of critical contingency events results in a violation of network constraints or customers not being supplied. However, before the distribution network operator can perform a security assessment or take required control actions (such as adjusting transformer taps, switching capacitor banks on or off, etc. [18]–[20]), accurate information about the state of the network must be obtained [21]. This information is provided by the state estimation process. State estimation is a standard procedure in transmission networks. However, due to the fundamental differences between distribution and transmission networks, the traditional appraoch to state estimation cannot be applied to distribution networks directly. Transmission networks connect to large central generators at a high voltage level in order to reduce losses while transmitting electric power over long distances. They are also strongly interconnected and equipped with a large number of redundant measurements in order to increase the reliability of the system. Distribution networks on the other hand operate at a medium to low voltage level, and function as the connection between the transmission network substations and the consumers. They are generally operated in a radial or lightly mashed configuration and, compared to the transmission systems, contain a very small number of measurement devices. Due to the large number of redundant measurement devices installed throughout transmission networks it is possible to estimate the state of a transmission network with a high degree of accuracy. This is, however, not the case in distribution networks where only a very small number of real-time measurements are available. In order to establish observability of a distribution network it is generally necessary to use a large number of pseudo-measurements. Because they are based on historical data, pseudo-measurements typically have large margins of error associated with them. The uncertainty introduced into the state estimation by pseudo-measurements is generally not an issue in transmission systems because the state estimator is able to filter out measurement errors due to the generally large number of redundant measurements [22], [23]. However, in distribution networks this is only possible to a very limited degree, if at all, due to the low number of real-time measurements. Hence, if a network is made observable by using a large number of pseudo-measurements, the estimated state might be significantly different from the actual network state even though the network is still classified as observable [24]. For this reason, state estimation has not been widely implemented in distribution networks to date [25].

1.4 Meter Placement

If the current measurement set of a network is insufficient to provide the required level of redundancy and/or accuracy, additional measurement devices need to be placed in the network. In transmission systems, additional measurement devices are normally placed in order to increase redundancy rather than state estimation accuracy, because the accuracy is generally high anyway due to the large number of available real-time measurements. In distribution networks, however, the number of available measurements is low which means that redundancy is normally not available. Therefore, redundancy is also not a consideration for the placement of additional measurement devices in distribution networks. Hence, in distribution networks additional measurement devices are generally placed in order to increase the accuracy of the state estimation result [26], [27]. However, due to economical constraints and the large number of buses, measurement devices cannot be placed at every bus in the network. This leads to the problem of optimal meter placement in distribution networks.

1.5 Summary

In summary, state estimation is a well known and established technique in transmission networks. It is used to obtain an estimate of the network state from the available set of measurements by using a maximum likelihood estimation. This means that it produces the most likely state of the network given the available measurement values and their associated measurement accuracy values. Historically there was no need for an on-line application of state estimation in distribution networks. However, due to the increase in renewable generation connected at the distribution level has increased the risk that network constraints are violated. This situation which gave rise to the need of an efficient on-line network management. The core of this management system is the state estimator. However in distribution networks the situation is different and the lack of measurements make an application of the traditional appraoch to state estimation difficult.

1.6 Project objectives

The objective of this thesis is to develop new methods that support an application of state estimation at the distribution level without the need for a large number of redundant measurements to be available. More specifically:

- To develop a practical appraoch to state estimation which does not depend on a large number of measurement devices in order to provide practical results.
- To develop a method which can identify network where it is required to apply a three phase state estimation in order to achieve a desired state estimation accuracy.
- To develop a method to calculate the probability density functions of the estimated parameters for an on-line application.
- To develop a probabilistic approach to observability that is suitable for an application in distribution networks.
- To develop a meter placement method which make an application of state estimation in distribution networks more practical.

Chapters 2-4 has been removed for copyright or proprietary reasons.

Published papers are located within these chapters. See thesis pages ii-iii and vii-viii

Chapter 5

Project Outcomes and Future Work

5.1 Project Outcomes

The aim of this thesis has been to develop new methods in order to support an application of state estimation in distribution networks without requiring a large number of real-time measurements. The main disadvantage of the traditional approach to state estimation is that it attempts to estimate the exact value of the network state parameters, and therefore, does not provide any information about the accuracy of the estimated state. This works well if the estimate has a high degree of accuracy which is, however, generally not the case in distribution systems. Hence, a new approach to state estimation in distribution networks has been developed as part of this thesis. The main idea of this approach is that the distribution network operator does not need to know the exact value of the estimated parameters but rather if these parameters are within their respective constraints. This is achieved by providing the confidence that the estimated parameters are within their respective constraints. As a result it is possible to combine information about the proximity of the estimated state to its constraints as well as its accuracy into a single number. It was shown that the confidence values are easy to interpret and are able to provide practical information about the network state even in presence of uncertain state estimation inputs. This approach can be used to implement state estimation in networks where state estimation was previously not practical due to the low number of real-time measurements.

Apart from the low number of real-time measurements, the problem of unbalance can be a source of uncertainty in distribution networks if a single phase state estimator is used. In the transmission networks it can generally be assumed that the system is balanced. However, this is not always the case in distribution networks. Hence, depending on the amount of unbalance this assumption can reduce the accuracy of the estimated network state. As part of this thesis a new method has been developed which is able to determine if a single-phase state estimator is sufficient to estimate the state of a given distribution network with a specified degree of accuracy, or if a three-phase state estimator is required. This has the advantage that a more complex and computationally demanding three-phase state estimator is only used if a desired degree of state estimation accuracy cannot be achieved with a single phase state estimator or if it would require a number of additional measurement devices that is not economical. Therefore, the proposed method can potentially reduce the computation time required for the state estimation and/or the cost of the required computing infrastructure compared to a network wide application of a three-phase state estimator.

In order to assess the accuracy of a network state, the probability density functions of the estimated parameters has to be available. The methods proposed in this thesis require all information about the accuracy of the parameter estimates in form of the probability density function. Especially for on-line applications computationally efficient methods have to be used to calculate the probability density functions of the estimated parameters. However, for some parts of a networks it may be necessary to use a computationally more demanding approach due to the specific network and/or measurement configuration. Hence, a method has been developed which is able to compute the probability density function of the estimated parameters on-line, without limitations in terms of network and/or measurement configurations as long as it is possible to estimate the network state. This method is designed to be applied to small areas of the network where other computationally efficient methods cannot be applied.

Since the number of real-time measurements is very low compared to the number of buses in distribution networks, distribution networks are generally unobservable without the use of pseudo measurements. This lack of measurements can always be compensated by adding pseudo measurements. However, due to the large amount of uncertainty associated with pseudo measurements, the estimated network state could be significantly different from the actual networks state, even if the network is classified as observable. For this reason a new probabilistic approach to distribution networks observability has been introduced that identifies critical areas in the system based on their state estimation accuracy and proximity to the constraints under worst case consideration. As part of the work on distribution network observability, in this thesis a new index has been proposed which is able to provide an indication about the network observability by quantifying the uncertainty in the state estimation result. This approach can also be used to compare different network state or measurement configurations with each other, with respect of their impact on the overall system uncertainty.

If a given measurement configuration is not sufficient to provide practical results, additional measurement devices have to be installed in the network. However, due to economical constraints only a limited number of additional measurement devices can be placed. This means that the best position for the additional measurement devices as well as how many measurement devices are required to deliver practical state estimation results has to be determined. For this purpose a new method has been developed which is able to achieve a practical level of state estimation accuracy while potentially reducing the number of placed measurements. The case studies have shown that the proposed method can result in a lower number of required measurement devices compared to the conventional methods. This indicates that the proposed method can lead to significant cost savings without loss of practicality.

In summary, this thesis makes the following contributions:

- A practical appraoch to state estimation based on confidence which is able to provide practical results in presence of uncertain state estimation inputs.

- A new method which can determine if a three phase state estimation is required for a specific distribution network based on the network uncertainty, desired accuracy, as well as the measurement and network configuration.
- A new computationally efficient method to calculate the probability density functions of estimated parameters which is suitable for on-line applications. This method can be applied to parts of the distribution network where other computationally efficient methods could previously not be applied due to the specific measurement and/or network configuration.
- A probabilistic approach to distribution network observability which takes the accuracy of the state estimation result into account and identifies critical areas.
- A method that quantifies the observability of a state estimation result using information entropy. This method provides a single number representation of the state estimation uncertainty which can be used to asses the observability of a network and to compare different network states or measurement configurations with each other.
- A new approach to meter placement in distribution networks based on practical considerations. The advantage of this method is that it does only place additional measurement devices in order to increase the accuracy of parameters that are critical, resulting in a smaller number of measurements placed while maintaining a practical level of state estimation accuracy.

5.2 Future Work

A logical next step to continue this research would be to verifying the applicability and practicality of the proposed methods on a larger real distribution system over a period that covers the four seasons in order to account for changes in temperature, irradiance, customer habits etc. The system would require a relatively large number of real-time measurements in order to verify the results of the proposed methods.

Another interesting research avenue would be to extend the developed measurement placement method to unbalanced systems which use a three phase state estimator. This work should also consider different measurement types such as current magnitudes, power flows, and phasor measurement units (PMUs) with respect to their cost per unit as well as contribution to the state estimation accuracy.

References

[1] F. C. Schweppe and J. Wildes, "Power system static-state estimation, part i: Exact model," *IEEE T. Power. Ap. Syst.*, vol. PAS-89, no. 1, pp. 120–125,

[2] F. Schweppe and D. Rom, "Power system static-state estimation, part II: Approximate model," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-89, no. 1, pp. 125–130, 1970.

[3] F. Schweppe, "Power system static-state estimation, part III: Implementation," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-89, no. 1, pp. 130–135, 1970.

[4] J. J. Grainger and W. D. Stevenson, *Power system analysis*, ser. McGraw-Hill series in electrical and computer engineering. New York: McGraw-Hill, International ed., 1994.

[5] A. Monticelli and F. F. Wu, "Network observability theory," *IEEE T. Power. Ap. Syst.*, vol. Vol.PAS-104, no. 5, pp. 1042–1048,

[6] R. Gelagaev, P. Vermeyen, J. Vandewalle, and J. Driesen, "Numerical observability analysis of distribution systems," in *14th Int. Conf. on Harmonics and Quality of Power (ICHQP)*, pp. 1–6.

[7] K. Clanents, G. Krutnpholz, and P. Davis, "Power system state estimation with measurement deficiency: An observability/measurement placement algorithm," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, no. 7, pp. 2012–2020, 1983.

[8] G. R. Krumpholz, K. A. Clements, and P. W. Davis, "Power system observability: A practical algorithm using network topology," *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 4, pp. 1534–1542,

[9] A. Monticelli and F. F. Wu, "Observability analysis for orthogonal transformation based state estimation," *IEEE Transactions on Power Systems*, vol. 1, no. 1, pp. 201–206, 1986.

[10] E. Manitsas, R. Singh, B. Pal, and G. Strbac, "Modelling of pseudo-measurements for distribution system state estimation," in *SmartGrids for Distribution, 2008. IET-CIRED. CIRED Seminar*, pp. 1–4.

[11] R. Singh, B. C. Pal, and R. A. Jabr, "Choice of estimator for distribution system state estimation," *Generation, Transmission & Distribution, IET*, vol. 3, no. 7, pp. 666–678, 2009.

[12] A. Abur and A. G. Expósito, *Power system state estimation theory and implementation.* New York: Marcel Dekker, Inc., 2004.

[13] *Australian pv institue. accessed: (23. oct. 2015). available: Www.apvi.org.au.*

[14] *Installed soalr capacity in australia. accessed: (11 september 2017). available: Http://pv-map.apvi.org.au/analyses.*

[15] D. Echternacht, M. Franken, A. Moser, U. Geulen, P. Zimmer, W. Feilhauer, and W. Schrieder, "Smart area aachen - in field test of meter placement and state estimation algorithms for distribution grids," in *Innovative Smart Grid Technologies Latin America (ISGT LATAM), 2015 IEEE PES*, pp. 435–439.

[16] W. El-Khattam, Y. G. Hegazy, and M. M. A. Salama, "Investigating distributed generation systems performance using monte carlo simulation," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 524–532, 2006.

[17] N. Etherden and M. H. J. Bollen, "Increasing the hosting capacity of distribution networks by curtailment of renewable energy resources," in *2011 IEEE Trondheim PowerTech*, 2011, pp. 1–7.

[18] L. Ramesh, S. P. Chowdhury, S. Chowdhury, a. a. Natarajan, Y. H. Song, and P. K. Goswami, "Distributed state estimation technique for active distribution networks," in *42nd International Universities Power Engineering Conference*, Ieee, pp. 861–866.

[19] C. M. Hird, H. Leite, N. Jenkins, and H. Li, "Network voltage controller for distributed generation," in *Generation, Transmission and Distribution*, vol. 151, pp. 150–156.

[20] F. Jiyuan and S. Borlase, "The evolution of distribution," *IEEE, Power and Energy Magazine*, vol. 7, no. 2, pp. 63–68, 2009.

[21] S. Raghuraman and R. Jegatheesan, "A survey on state estimation techniques in electrical power system," *2011 International Conference on Recent Advancements in Electrical, Electronics and Control Engineering*, pp. 199–205, 2011.

[22] M. E. Baran and A. W. Kelley, "State estimation for real-time monitoring of distribution systems," *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1601–1609, 1994.

[23] A. K. Al-Othman and M. R. Irving, "Uncertainty modelling in power system state estimation," *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 152, no. 2, pp. 233–239, 2005.

[24] L. Ke, "State estimation for power distribution system and measurement impacts," *IEEE Trans. on Power Syst.*, vol. 11, no. 2, pp. 911–916,

[25] R. Singh, B. C. Pal, and R. A. Jabr, "Statistical representation of distribution system loads using gaussian mixture model," *IEEE Transactions on Power Systems*, vol. 25, no. 1, pp. 29–37, 2010.

[26] L. Junqi, F. Ponci, A. Monti, C. Muscas, P. A. Pegoraro, and S. Sulis, "Optimal meter placement for robust measurement systems in active distribution grids," *IEEE Trans. Instrum. Meas.*, vol. 63, no. 5, pp. 1096–1105, 2014.

[27] N. Nusrat, M. Irving, and G. Taylor, "Novel meter placement algorithm for enhanced accuracy of distribution system state estimation," in *IEEE Power and Energy Society General Meeting*, Institute of Electrical and Electronics Engineers (IEEE), 2012, pp. 1–8.

[28] R. C. Dugan, R. F. Arritt, T. E. McDermott, S. M. Brahma, and K. Schneider, "Distribution system analysis to support the smart grid," in *IEEE PES General Meeting*, pp. 1–8.

[29] I. Roytelman and S. Shahidehpour, "State estimation for electric power distribution systems in quasi real-time conditions," *IEEE Transactions on Power Delivery*, vol. 8, no. 4, pp. 2009–2015, 1993.

[30] W.-M. Lin and J.-H. Teng, "State estimation for distribution systems with zero-injection constraints," *IEEE Transactions on Power Systems*, vol. 11, no. 1, pp. 518–524, 1996.

[31] C. Xu and A. Abur, "Robust linear state estimation with equality constraints," in *2016 IEEE Power and Energy Society General Meeting (PESGM)*, 2016, pp. 1–5.

[32] O. Chilard, S. Grenard, O. Devaux, and L. de Alvaro Garcia, "Distribution state estimation based on voltage state variables : Assessment of results and limitations," in *CIRED 2009 - 20th International Conference and Exhibition on Electricity Distribution - Part 1*, 2009, pp. 1–4.

[33] C. Hansen and A. Debs, "Power system state estimation using three-phase models," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 818–824, 1995.

[34] D. A. Haughton and Heydt, "A linear state estimation formulation for smart distribution systems," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1187–1195, 2013.

[35] P. Chusovitin, I. Polyakov, and A. Pazderin, "Three-phase state estimation model for distribution grids," in *2016 IEEE International Conference on the Science of Electrical Engineering (ICSEE)*, Institute of Electrical and Electronics Engineers (IEEE), 2016.

[36] D. Rusinaru, L. G. Manescu, M. Ciontu, and M. Alba, “Three-phase load flow analysis of the unbalanced distribution networks,” in *2016 International Conference on Applied and Theoretical Electricity (ICATE)*, Institute of Electrical and Electronics Engineers (IEEE), 2016.

[37] A. Garces, “A linear three-phase load flow for power distribution systems,” *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 827–828, 2016.

[38] H. Ahmadi, J. R. Marti, and A. von Meier, “A linear power flow formulation for _newline three-phase distribution systems,” *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 5012–5021, 2016.

[39] M. Baran and A. Kelley, “A branch-current-based state estimation method for distribution systems,” *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 483–491, 1995.

[40] W. M. Lin and J. H. Teng, “Distribution fast decoupled state estimation by measurement pairing,” *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 143, no. 1, pp. 43–48, 1996.

[41] W.-M. Lin, J.-H. Teng, and S.-J. Chen, “A highly efficient algorithm in treating current measurements for the branch-current-based distribution state estimation,” *IEEE Transactions on Power Delivery*, vol. 16, no. 3, pp. 433–439, 2001.

[42] C. Muscas, S. Sulis, A. Angioni, F. Ponci, and A. Monti, “Impact of different uncertainty sources on a three-phase state estimator for distribution networks,” *IEEE Transactions on Instrumentation and Measurement*, vol. 63, no. 9, pp. 2200–2209, 2014.

[43] C. Muscas, M. Pau, P. A. Pegoraro, S. Sulis, F. Ponci, and A. Monti, “Multiarea distribution system state estimation,” *IEEE Trans. Instrum. Meas.*, vol. 64, no. 5, pp. 1140–1148, 2015.

[44] C. Muscas, P. A. Pegoraro, S. Sulis, M. Pau, F. Ponci, and A. Monti, “Fast multi-area approach for distribution system state estimation,” in *2016 IEEE International Instrumentation and Measurement Technology Conference Proceedings*, pp. 1–6.

[45] Z. Liang and A. Abur, "Multi area state estimation using synchronized phasor measurements," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 611–617, 2005.

[46] A. Ghosh, D. Lubkeman, M. Downey, and R. Jones, "Distribution circuit state estimation using a probabilistic approach," *IEEE Trans. Power Syst.*, vol. 12, no. 1, pp. 45–51, 1997.

[47] E.-H. M. Bernd Brinkmann Michael Negnevitsky, *Advances in electric power and energy systems: Power systems engineering*, ser. IEEE Press Series on Power Engineering. Wiley.

[48] B. Brinkmann, M. Negnevitsky, T. Yee, and T Nguyen, "An observability index for distribution networks using information entropy," in *Australasian Universities Power Engineering Conf. (AUPEC)*, pp. 1–6.

[49] B. Brinkmann and M. Negnevisky, "Robust state estimation in distribution networks," in *2016 Australasian Universities Power Engineering Conference (AUPEC)*, pp. 1–5.

[50] M. Strelec, P. Janecek, D. Georgiev, A. Zapotocka, and E. Janecek, "Backward/forward probabilistic network state estimation tool and its real world validation," in *Power and Electrical Engineering of Riga Technical University (RTUCON), 2015 56th International Scientific Conference on*, pp. 1–6.

[51] G. Valverde, A. T. Saric, and V. Terzija, "Stochastic monitoring of distribution networks including correlated input variables," *IEEE Trans. on Power Syst.*, vol. 28, no. 1, pp. 246–255, 2013.

[52] L. Guang, Z. Ning, T. Ferryman, and F. Tuffner, "Uncertainty quantification in state estimation using the probabilistic collocation method," in *IEEE/PES Power Systems Conference and Exposition (PSCE)*, pp. 1–8.

[53] G. M. Mathews, "The accuracy of factored nonlinear weighted least squares state estimation," in *IEEE Int. Energy Conf. and Exhibition (ENERGYCON)*, pp. 860–866.

[54] D. Lubkeman, J. Zhang, A. Ghosh, and R. Jones, "Field results for a distribution circuit state estimator implementation," *IEEE Transactions on Power Delivery*, vol. 15, no. 1, pp. 399–406, 2000.

[55] *Tasmanian electricity code accessed: (28. april. 2015). available: Http://www.energyregula v/003.*

[56] *Queensland electricity regulation accessed: (25. feb. 2016). available: Https://www.legislati*

[57] A. C. Melhorn and A. Dimitrovski, "A method for modeling voltage regulators in probabilistic load flow for radial systems," in *International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, pp. 1–6.

[58] F. Shabani, N. R. Prasad, and H. A. Smolleck, "State estimation with aid of fuzzy logic," in *Proceedings of the Fifth IEEE International Conference on Fuzzy Systems*, vol. 2, 947–953 vol.2.

[59] "Ieee pes radial distribution test feeders. accessed: (05 august 2013). available: Http://ewh.ieee.org/soc/pes/dsacom/testfeeders/,"

[60] B. Das, "Consideration of input parameter uncertainties in load flow solution of three-phase unbalanced radial distribution system," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1088–1095, 2006.

[61] A. Majumdar and B. C. Pal, "A three-phase state estimation in unbalanced distribution networks with switch modelling," in *2016 IEEE First International Conference on Control, Measurement and Instrumentation (CMI)*, pp. 474–478.

[62] R. Yan and T. K. Saha, "Voltage variation sensitivity analysis for unbalanced distribution networks due to photovoltaic power fluctuations," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 1078–1089, 2012.

[63] N. M. Manousakis, G. N. Korres, S. Member, and P. S. Georgilakis, "Taxonomy of pmu placement methodologies," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 1070–1077, 2012.

[64] B. Brinkmann and M. Negnevitsky, "A probabilistic approach to observability of distribution networks," *IEEE Trans. on Power Syst.*, pp. 1–10, 2016.

[65] M. O. W. Grond, H. N. Luong, J. Morren, P. A. N. Bosman, H. J. G. Slootweg, and H. La Poutre, "Practice-oriented optimization of distribution network planning using metaheuristic algorithms," in *Power Syst. Computation Conf. (PSCC)*, pp. 1–8.

[66] V. K. Rohatgi, A. K.M. E. Saleh, and V. K. Rohatgi, *An introduction to probability and statistics*, ser. Wiley series in probability and statistics: Texts and references section. New York : Wiley, 2001.

[67] R. Singh, B. C. Pal, R. A. Jabr, and R. B. Vinter, "Meter placement for distribution system state estimation: An ordinal optimization approach," *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 2328–2335, 2011.

[68] R. Singh, B. C. Pal, and R. B. Vinter, "Measurement placement in distribution system state estimation," *IEEE Trans. on Power Syst.*, vol. 24, no. 2, pp. 668–675,

[69] E. Manitsas, R. Singh, B. C. Pal, and G. Strbac, "Distribution system state estimation using an artificial neural network approach for pseudo measurement modeling," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1888–1896,

[70] A. Gomez-Exposito and A. Abur, "Generalized observability analysis and measurement classification," *IEEE Trans. on Power Syst.*, vol. 13, no. 3, pp. 1090–1095,

[71] H. Mori and S. Tsuzuki, "A fast method for topological observability analysis using a minimum spanning tree technique," *IEEE Transactions on Power Systems*, vol. 6, no. 2, pp. 491–500, 1991.

[72] A. Costa, E. Lourenco, and K. Clements, "Power system topological observability analysis including switching branches," *IEEE Transactions on Power Systems*, vol. 17, no. 2, pp. 250–256, 2002.

[73] H. Mori, "A ga-based method for optimizing topological observability index in electric power networks," in *IEEE World Congress on Computational Intelligence*, pp. 565–568.

[74] A. Monticelli, "The impact of modeling short circuit branches in state estimation," *IEEE Transactions on Power Systems*, vol. 8, no. 1, pp. 364–370, 1993.

[75] P. Katsikas and G. Korres, "Unified observability analysis and measurement placement in generalized state estimation," *IEEE Transactions on Power Systems*, vol. 18, no. 1, pp. 324–333, 2003.

[76] T. Baldwin, L. Mili, M. Boisen, and R. Adapa, "Power system observability with minimal phasor measurement placement," *IEEE Transactions on Power Systems*, vol. 8, no. 2, pp. 707–715, 1993.

[77] M. Göl, S. Member, and A. Abur, "Observability and criticality analyses for power systems measured by phasor measurements," vol. 28, no. 3, pp. 3319–3326, 2013.

[78] G. Korres and N. Manousakis, "State estimation and observability analysis for phasor measurement unit measured systems," *IET Generation, Transmission & Distribution*, vol. 6, no. 9, p. 902, 2012.

[79] J. Jerome, "Network observability and bad data processing algorithm for distribution networks," vol. 00, no. C, pp. 1692–1697, 2001.

[80] D. Toyoshima, M. R. C. Castillo, C. A. Fantin, and J. B. A. London, "Observability and measurement redundancy analysis on three-phase state estimation," in *2012 IEEE Power and Energy Society General Meeting*, IEEE, 2012.

[81] F. Magnago, L. Zhang, and M. K. Celik, "Multiphase observability analysis in distribution systems state estimation," in *2016 Power Systems Computation Conference (PSCC)*, Institute of Electrical and Electronics Engineers (IEEE), 2016.

[82] M. G. Damavandi, V. Krishnamurthy, and J. R. Marti, "Robust meter placement for state estimation in active distribution systems," *IEEE Transactions on Smart Grid*, vol. 6, no. 4, pp. 1972–1982, 2015.

[83] A. Kulmala, S. Repo, and P. Jarventausta, "Using statistical distribution network planning for voltage control method selection," in *Renewable Power Generation Conf.*, pp. 1–6.

[84] M. O. W. Grond, J. Morren, and J. G. Slootweg, "Integrating smart grid solutions into distribution network planning," in *PowerTech (POWERTECH), 2013 IEEE Grenoble*, pp. 1–6.

[85] G. Celli, F. Pilo, G. G. Soma, R. Cicoria, G. Mauri, E. Fasciolo, and G. Fogliata, "A comparison of distribution network planning solutions: Traditional reinforcement versus integration of distributed energy storage," in *IEEE PowerTech (POWERTECH)*, pp. 1–6.

[86] M. Dejun, W. Yong, T. Xiuxia, H. Liangliang, H. Zongshuai, and F. Rusen, "Least squares class power system state estimation algorithm efficiency analysis," in *Technological Advances in Electrical, Electronics and Computer Engineering (TAEECE), 2015 Third International Conference on*, pp. 17–21.

[87] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal, The*, vol. 27, no. 3, pp. 379–423,

[88] X. Li, J. Yao, X. Liu, and H. Guan, "A first look at information entropy-based data pricing," in *2017 IEEE 37th International Conference on Distributed Computing Systems (ICDCS)*, 2017, pp. 2053–2060.

[89] G. Xiantai, M. Shiheng, J. Weidong, and L. Xiao, "Fault type recognition of high-speed train bogie based on dual-channel integration of information entropy," in *2016 2nd IEEE International Conference on Computer and Communications (ICCC)*, 2016, pp. 1880–1884.

[90] M. Bichsel and P. Seitz, "Minimum class entropy: A maximum information approach to layered networks," *Neural Networks*, vol. 2, no. 2, pp. 133–141, 1989.

[91] P. A. Pegoraro and S. Sulis, "On the uncertainty evaluation in distribution system state estimation," in *Smart Measurements for Future Grids (SMFG), 2011 IEEE International Conference on*, pp. 59–63.

[92] G. D'Antona and M. Davoudi, "Effects of parameter and measurement uncertainties on the power system wls state estimation," in *Instrumentation and Measurement Technology Conference (I2MTC), 2012 IEEE International*, IEEE, 2012, pp. 1015–1020.

[93] M. Negnevitsky, J. Terry, and N. Thanh, "Using information entropy to quantify uncertainty in distribution networks," in *Australasian Universities Power Engineering Conference (AUPEC)*, pp. 1–6.

[94] X. Chen, J. Lin, C. Wan, Y. Song, S. You, Y. Zong, W. Guo, and Y. Li, "Optimal meter placement for distribution network state estimation: A circuit representation based MILP approach," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 4357–4370, 2016.

[95] D. Echternacht and A. Moser, "Cost optimal meter placement in low and medium voltagte grids considering stochastic dependencies," in *IEEE PES ISGT Europe 2013*, Institute of Electrical and Electronics Engineers (IEEE), 2013.

[96] L. Junqi, T. Junjie, F. Ponci, A. Monti, C. Muscas, and P. A. Pegoraro, "Trade-offs in pmu deployment for state estimation in active distribution grids," *IEEE Transactions on Smart Grid*, vol. 3, no. 2, pp. 915–924, 2012.

Appendix

5.3 Appendix A

The 145 bus test feeder is located in Queensland which is a state in Australia. It services a rural area with a nominal voltage of 11kV. The topology of the feeder is shown in Fig. 5.1 where the square represents the slack bus, the circle represents a voltage regulator and the two arrows indicate power flow measurements. The voltage regulator has a set voltage of 1 p.u. and a bandwidth of ±0.009 p.u. The power flow measurements have an accuracy of ±10%.

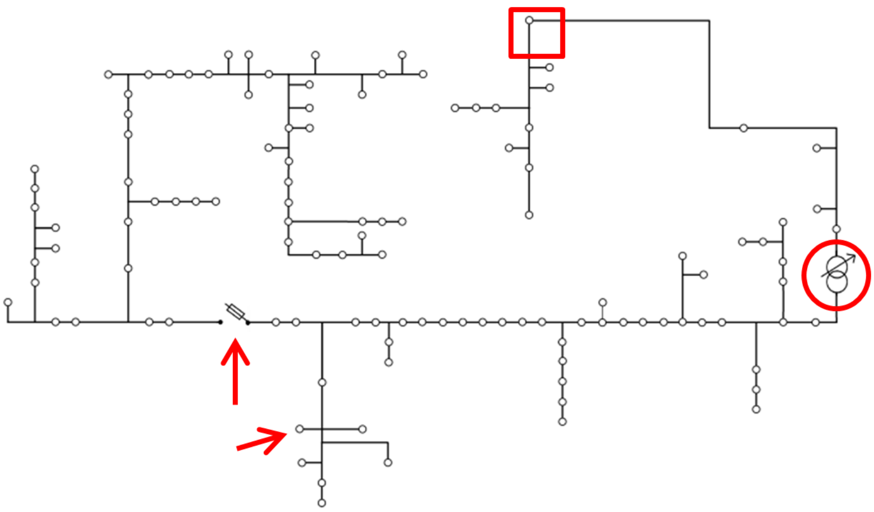

Figure 5.1: The 145 bus test feeder topology

The line data of the feeder is given in the table below.

Table 5.1: 145 bus feeder line data

From Bus	To Bus	R	X	From Bus	To Bus	R	X
1	2	0.0908	0.0675	69	74	0.0959	0.0966
2	3	1.7193	0.1122	64	75	0.2718	0.2752
2	4	0.0683	0.0207	75	76	0.2481	0.2512
4	5	0.1784	0.0542	76	77	0.0291	0.0123
4	6	0.2233	0.0678	77	78	0.2241	0.0943
6	7	0.5197	0.1578	78	79	0.1706	0.0718
7	8	1.6692	0.1089	79	80	0.1523	0.0641
8	9	2.6857	0.1753	80	81	1.5060	0.0983
6	10	0.2925	0.0888	81	82	2.5100	0.1638
10	11	0.4875	0.1480	82	83	3.7399	0.2441
11	12	0.2525	0.0767	83	84	3.7399	0.2441
11	13	0.2038	0.0619	83	85	4.6560	0.3038
13	14	0.5889	0.1788	85	86	2.6983	0.1761
1	15	1.0653	1.1164	86	87	1.2550	0.0819
15	16	0.2618	0.2639	87	88	1.5437	0.1007
16	17	0.0844	0.0848	87	89	2.5853	0.1687
16	18	0.4880	0.4902	89	90	2.2967	0.1499
18	19	1.6440	0.1073	89	91	1.2424	0.0811
18	20	0.0696	0.0765	91	92	5.4341	0.3546
20	21	0.2479	0.2686	92	93	2.7359	0.1785
21	22	0.2733	0.2768	80	94	0.1185	0.0499
22	23	0.1627	0.1648	94	95	0.1212	0.0510
23	24	0.2544	0.2576	95	96	0.2268	0.0955
24	25	0.2323	0.2352	96	97	2.8363	0.1851
25	26	0.2481	0.2512	97	98	5.1079	0.3333
26	27	0.2007	0.2032	98	99	4.8819	0.3186
25	28	0.3097	0.3136	99	100	2.1460	0.1400
22	29	0.0541	0.0544	96	101	0.0508	0.0214

Table 5.1: 145 bus feeder line data

From Bus	To Bus	R	X	From Bus	To Bus	R	X
29	30	0.5988	0.6064	101	102	0.3223	0.1357
30	31	0.2623	0.2656	102	103	0.1401	0.0590
31	32	0.3271	0.3312	103	104	0.0677	0.0285
29	33	0.0377	0.0378	104	105	0.0677	0.0285
33	34	0.2670	0.2704	105	106	0.0243	0.0618
34	35	0.2860	0.2896	105	107	0.0988	0.0416
35	36	0.5103	0.5168	107	108	0.2173	0.0915
36	37	0.0363	0.0368	108	109	0.2444	0.1029
36	38	0.4424	0.4480	109	110	0.2464	0.1037
35	39	0.5135	0.5200	110	111	0.0542	0.0228
39	40	0.4535	0.4592	111	112	0.0068	0.0058
40	41	0.5230	0.5296	111	113	0.0325	0.0137
41	42	0.3113	0.3152	113	114	1.4683	0.0958
42	43	0.7647	0.7744	113	115	2.3092	0.1507
42	44	0.4566	0.4624	113	116	0.5530	0.5600
44	45	0.3650	0.3696	116	117	0.2370	0.2400
45	46	0.1469	0.1488	117	118	0.1217	0.1232
46	47	0.5546	0.5616	118	119	0.7284	0.7376
47	48	0.1501	0.1520	118	120	0.1580	0.1600
48	49	0.1375	0.1392	120	121	0.2212	0.2240
49	50	0.1359	0.1376	120	122	0.7078	0.7168
45	51	0.4282	0.4336	122	123	0.1580	0.1600
51	52	0.1311	0.1328	123	124	0.1185	0.1200
52	53	0.2639	0.2672	123	125	1.0191	1.0320
53	54	0.1991	0.2016	117	126	0.0190	0.0080
54	55	0.0806	0.0816	117	127	0.0174	0.0176
55	56	0.2765	0.2800	127	128	1.3052	0.0852
56	57	0.1248	0.1264	127	129	0.3618	0.3664

Table 5.1: 145 bus feeder line data

From Bus	To Bus	R	X	From Bus	To Bus	R	X
57	58	0.2544	0.2576	129	130	0.2891	0.2928
58	59	0.2765	0.2800	129	131	0.1754	0.1776
59	60	0.1343	0.1360	131	132	0.2481	0.2512
60	61	0.3950	0.4000	131	133	0.1106	0.1120
59	62	0.1011	0.1024	133	134	0.1217	0.1232
62	63	0.2733	0.2768	134	135	0.1438	0.1456
63	64	0.5751	0.5824	135	136	0.1580	0.1600
64	65	0.4266	0.4320	136	137	0.2544	0.2576
65	66	0.1659	0.1680	137	138	0.0664	0.0672
66	67	0.2038	0.2064	138	139	0.3754	0.1140
66	68	0.1596	0.1616	139	140	0.0975	0.0296
66	69	0.1327	0.1344	140	141	0.3900	0.1184
69	70	0.0995	0.1008	140	142	0.2808	0.0852
70	71	0.1296	0.1312	136	143	0.4021	0.1693
70	72	0.6320	0.6400	143	144	0.3696	0.1556
72	73	1.9797	2.0048	144	145	0.2559	0.1077

The load data for the high load case without distributed generation form PV installations (due to lack of sunshine) is given in the table below.

Table 5.2: 145 bus feeder load data

Bus	P[kw]	Bus	Q [kvar]	Bus	P[kw]	Bus	Q [kvar]
2	0.000	2	0.000	74	1.131	74	0.330
3	1.131	3	0.330	75	13.572	75	3.958
4	0.000	4	0.000	76	4.524	76	1.319
5	1.131	5	0.330	77	0.000	77	0.000
6	0.000	6	0.000	78	10.179	78	2.969
7	2.262	7	0.660	79	3.393	79	0.990

Table 5.2: 145 bus feeder load data

Bus	P[kw]	Bus	Q [kvar]	Bus	P[kw]	Bus	Q [kvar]
8	1.131	8	0.330	80	0.000	80	0.000
9	1.131	9	0.330	81	1.131	81	0.330
10	1.131	10	0.330	82	2.262	82	0.660
11	0.000	11	0.000	83	0.000	83	0.000
12	1.131	12	0.330	84	4.524	84	1.319
13	5.655	13	1.649	85	2.262	85	0.660
14	1.131	14	0.330	86	1.131	86	0.330
15	0.000	15	0.000	87	0.000	87	0.000
16	0.000	16	0.000	88	2.262	88	0.660
17	1.131	17	0.330	89	0.000	89	0.000
18	0.000	18	0.000	90	4.524	90	1.319
19	1.131	19	0.330	91	1.131	91	0.330
20	1.131	20	0.330	92	1.131	92	0.330
21	6.786	21	1.979	93	1.131	93	0.330
22	2.262	22	0.660	94	2.262	94	0.660
23	5.655	23	1.649	95	5.655	95	1.649
24	1.131	24	0.330	96	0.000	96	0.000
25	0.000	25	0.000	97	0.000	97	0.000
26	4.524	26	1.319	98	1.131	98	0.330
27	6.786	27	1.979	99	1.131	99	0.330
28	7.917	28	2.309	100	1.131	100	0.330
29	0.000	29	0.000	101	10.179	101	2.969
30	4.524	30	1.319	102	15.834	102	4.618
31	5.655	31	1.649	103	4.524	103	1.319
32	7.917	32	2.309	104	2.262	104	0.660
33	3.393	33	0.990	105	0.000	105	0.000
34	2.262	34	0.660	106	1.131	106	0.330
35	0.000	35	0.000	107	11.310	107	3.299

Table 5.2: 145 bus feeder load data

Bus	P[kw]	Bus	Q [kvar]	Bus	P[kw]	Bus	Q [kvar]
36	0.000	36	0.000	108	5.655	108	1.649
37	2.262	37	0.660	109	5.655	109	1.649
38	11.310	38	3.299	110	4.524	110	1.319
39	3.393	39	0.990	111	0.000	111	0.000
40	7.917	40	2.309	112	4.524	112	1.319
41	4.524	41	1.319	113	0.000	113	0.000
42	3.393	42	0.990	114	1.131	114	0.330
43	1.131	43	0.330	115	2.262	115	0.660
44	4.524	44	1.319	116	3.393	116	0.990
45	0.000	45	0.000	117	0.000	117	0.000
46	2.262	46	0.660	118	0.000	118	0.000
47	5.655	47	1.649	119	1.131	119	0.330
48	2.262	48	0.660	120	0.000	120	0.000
49	2.262	49	0.660	121	1.131	121	0.330
50	2.262	50	0.660	122	2.262	122	0.660
51	6.786	51	1.979	123	0.000	123	0.000
52	2.262	52	0.660	124	1.131	124	0.330
53	4.524	53	1.319	125	3.393	125	0.990
54	3.393	54	0.990	126	1.131	126	0.330
55	5.655	55	1.649	127	0.000	127	0.000
56	4.524	56	1.319	128	2.262	128	0.660
57	3.393	57	0.990	129	6.786	129	1.979
58	4.524	58	1.319	130	2.262	130	0.660
59	0.000	59	0.000	131	0.000	131	0.000
60	2.262	60	0.660	132	3.393	132	0.990
61	9.048	61	2.639	133	3.393	133	0.990
62	3.393	62	0.990	134	3.393	134	0.990
63	2.262	63	0.660	135	4.524	135	1.319

Table 5.2: 145 bus feeder load data

Bus	P[kw]	Bus	Q [kvar]	Bus	P[kw]	Bus	Q [kvar]
64	0.000	64	0.000	136	2.262	136	0.660
65	1.131	65	0.330	137	5.655	137	1.649
66	0.000	66	0.000	138	4.524	138	1.319
67	172.000	67	27.000	139	15.834	139	4.618
68	0.000	68	0.000	140	0.000	140	0.000
69	0.000	69	0.000	141	22.619	141	6.597
70	0.000	70	0.000	142	21.488	142	6.267
71	1.131	71	0.330	143	18.095	143	5.278
72	2.262	72	0.660	144	20.357	144	5.938
73	1.131	73	0.330	145	16.964	145	4.948